

# The County Fair Cyber Loss Distribution: Drawing Inferences from Insurance Prices

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Insurance premiums reflect expectations about the future losses of each insured. Given the dearth of cyber security loss data, market premiums could shed light on the true magnitude of cyber losses despite noise from factors unrelated to losses. To that end, we extract cyber insurance pricing information from the regulatory filings of 26 insurers. We provide empirical observations on how premiums vary by coverage type, amount, and policyholder type and over time. A method using *particle swarm optimisation* and the expected value premium principle is introduced to iterate through candidate parameterised distributions with the goal of reducing error in predicting observed prices. We then aggregate the inferred loss models across 6,828 observed prices from all 26 insurers to derive the *County Fair Cyber Loss Distribution*. We demonstrate its value in decision support by applying it to a theoretical retail firm with annual revenue of \$50M. The results suggest that the expected cyber liability loss is \$428K and that the firm faces a 2.3% chance of experiencing a cyber liability loss between \$100K and \$10M each year. The method and resulting estimates could help organisations better manage cyber risk, regardless of whether they purchase insurance.

CCS Concepts: • **Security and privacy** → **Economics of security and privacy**; • **Computing methodologies** → *Optimization algorithms*; • **Social and professional topics** → Economic impact;

Additional Key Words and Phrases: Quantifying cyber risk, cyber insurance, particle swarm optimization

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## 1 INTRODUCTION

Understanding how losses are distributed is an important step in assigning information security resources. The optimal investment level is intimately linked to both the likelihood and the impact of potential attacks. Put simply, “risks cannot be managed better until they can be measured better” [1].

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How can organisations gain insights into their distribution of cyber losses? Information sources include mandatory breach notifications, threat reports released by security vendors, self-reported survey data, and court dockets relating to cyber incidents. Potential insights are limited by challenges including the problem of denominators [2], reporting biases [3], and the tension between sample size and granularity.

The incentives for security vendors to misrepresent the threats faced have been widely recognised [4]. Self-reported survey results regarding cyber-crime costs are “dominated by a minority in the upper tail” and the effects of “refusal rates and small sample sizes” [3] are magnified by the rarity of reported cyber attacks. Court dockets are detailed but can only provide insights into costs assigned by courts.

Repositories of data breach reports, such as Privacy Rights Clearinghouse,<sup>1</sup> are drawn from the population of firms operating in the corresponding jurisdiction. This allows researchers to glean insights into the size of breaches and frequency in the aggregate. Estimating the frequency of incidents for an individual firm is complicated by not knowing the size of the population from which these reports are drawn [2].

Using the aforementioned data sources assumes homogeneity in the sample. This raises the question of how relevant the losses suffered by a \$300 billion energy firm are to a “mom-and-pop” retail outlet. But, throwing out such data reduces an already limited sample size. A novel data source for cyber loss distributions could be valuable to complement existing information sources, especially given their limitations.

Abramowicz argues the underwriting process by which insurers price and differentiate risks “can be translated into predictions” [5]. Abramowicz imagines insurance prices providing a rank ordering of how much litigation risk is associated with each policyholder. This ties into Hayek’s foundational work [6] on how price systems communicate information about “how to secure the best use of resources,” even when the “knowledge of the relevant facts is dispersed among many people.” Despite outlining the intuition that prices reflect dispersed information, neither author proposes a method of extracting quantitative estimates.

Prediction markets provide a way forward. A binary option on whether an event occurs is assumed to be a linear function of the expected probability of the event occurring. This has been used to predict election outcomes [7], influenza outbreaks [8], scientific results [9], Oscar nominees and winners [10], and government policy outcomes [11]. In the security context, it has been suggested that exploit derivatives can be used to estimate the probability a software product will be compromised [12] and that cyber warranties can be used to estimate expected cyber losses with the corresponding control in place [13].

This article proposes a method to extract information from insurance prices while acknowledging the role of market distortions. Our first contribution is to extract the first quantitative observations of cyber insurance prices across carriers, time, coverage amount, and insured firm characteristics. We show how prices fell as more insurers began offering cyber insurance with relevance to policy discussions [14].

Our second contribution is the introduction of a method to infer loss distributions from insurance prices, much like how probabilities are extracted from prediction markets. We adopt an iterative model fitting approach based on parsimonious assumptions about how actuaries translate loss expectations into insurance prices. Varying such assumptions provides a promising line of future work for actuarial science.

Our final contribution applies our new method to the observed prices, which leads to a diverse range of risk estimates. This is unsurprising as prices are noisy indicators that “incorporate falsehood as well as truth” [10]. Based on the intuition behind the wisdom of crowds [15], these estimates are aggregated to derive the *County Fair Cyber Loss Distribution*. Throughout we document the assumptions and discuss the limitations of this approach.

Using insurance prices in this way complements existing approaches to quantifying cyber losses. Estimates incorporate the granularity in cyber insurance pricing, which varies according to firm characteristics like revenue and industry. Insurance is also offered for many cyber incidents that currently lack related data. Finally, we provide full distributions of dollar losses, whereas previous studies report point-estimates of dollar losses or distributions of the number of records breached.

<sup>1</sup><https://www.privacyrights.org/>.

Table 1. Research Into the Cost and Frequency of Cyber Losses

Study	Year	Data Source	Data Points	Frequency	Impact
[16]	2008	Identity Theft Resource Center	899		
[17]	2010	Open Security Foundation DatalossDB	956		Power law*
[18]	2016	Privacy Rights Clearinghouse (PRC)	2,253		Lognormal*
[19]	2016	PRC & OPS DatalossDB &	8,574		Pareto*
[20]	2017	Privacy Rights Clearinghouse	2,266		Log-skew-normal*
[21]	2018	Privacy Rights Clearinghouse	600		Stochastic process*
[22]	2010	Research institute's event log	23,000		Lognormal*
[23]	2014	Nordic bank's event log	1,800		Lognormal*
[24]	2018	Proprietary data	53,308/2,216		
[25]	2018	Interviews	2,200		$\mu$
[26]	2015	Information security breaches survey	664	$\mu$	$\mu$
[27]	2018	Bank of Italy survey data	4,209		$\mu$
[28]	2014	US court dockets	230		
[29]	2017	UK ICO regulatory actions	118		$\mu$
[30]	2003	Event (DoS) window study	23		AR
[31]	2003	Event (Data Breach) window study	43		AR
[32]	2004	Event (Data Breach) window study	66		AR
[33]	2015	Operational risk database	994		$\mu, \sigma$
[34]	2018	ORX News	103		$\mu$
[35]	2019	SAS OpRisk Global data	1,579		$\mu, \sigma, MR$
[36]	2016	Proprietary data	12,585	$\mu$	MR

\* = Not financial impact,  $\mu$  = mean/mode/median,  $\sigma$  = variation, MR = Multivariable regression, AR = Abnormal returns.

Section 2 identifies research into the quantification of cyber losses at a firm level. Section 3 describes the pricing structure from all admitted insurers in California. We introduce our inference approach in Section 4. Section 5 applies the inference framework to evaluate how well these parameterised distributions explain observed data. We discuss limitations and compare our inferences to other empirical work related to loss events in Section 6. Finally, we offer conclusions in Section 7.

## 2 RELATED WORK

This section identifies existing approaches and techniques to quantify cyber losses. Table 1 provides an overview of the studies we have considered, including the data source of the study. The frequency and impact columns concern whether the study provides insights for individual organisations rather than statistics aggregated across multiple firms within an industry or economy.

Studying data breach repositories like Privacy Rights Clearinghouse provides insights into the size of breaches and aggregate frequency. Aggregate frequency is found to be stable over time [18, 19] and is distributed according to a “Poisson or negative binomial” [20]. Breach size was shown to be best described by a power law [17], Lognormal [18], Pareto [19], and log-skew-normal [20] distribution in successive publications. A recent paper has suggested both aggregate frequency and breach size are better modelled by stochastic processes [21].

Governments are the only entities who can realistically operationalise insights into aggregate frequency. Unfortunately they have a limited number of levers to pull in response. A notable exception is mandatory breach reporting laws, which have been shown to reduce identity theft by 6.1% [37]. Organisations have more risk management tools available, but how can they estimate potential losses?

Schroeder et al. [22] investigated 23,000 failures recorded on more than 20 different systems at a research institute. The results suggest time between failures is modelled by a Weibull distribution and “repair times are well modeled by a Lognormal distribution.” Franke et al. [23] investigated 1,800 incidents in a large Nordic bank and also found that the “Lognormal distribution offers the best fit of IT service time to recovery.” These studies do not provide incident costs. It is not clear how much we can generalise from atypical organisations like the Los Alamos National Laboratory [22].

Verizon’s *Data Breach Investigations Report* [24] describes the relative frequency of different types of incidents by industry. The Ponemon Institute’s *Cost of a Data Breach Report* [25] estimates the average cost per record in a data breach. However, Verizon [24] do not provide absolute frequencies and the Ponemon Institute [25] only surveys firms that have detected a breach. There are questions about the compatibility of commercial sponsors and scientific integrity, as evidenced by the \$1 trillion cyber crime figure.<sup>2</sup>

Surveys commissioned by governments provide an alternative. One study of security investments [26] used a survey commissioned by the UK Government [38] providing point-estimates of both frequency and impact distributed according to a Bernoulli distribution. Piggy-backing security-related questions on the Bank of Italy’s annual survey provided a larger set of responses [27]. Neither set of losses was fitted to a distribution. Furthermore, self-reported surveys are complicated by response biases [3].

Court dockets provide rich information about legal cases. Romanosky et al. [28] investigated factors affecting the data breach litigation. Freedom of information requests have been used to understand regulatory actions in the UK [29]. But they only provide insights into cases contested in the courts, leading to small sample sizes.

Event window studies provide insights into how events, such as denial of service attacks [30] and data breaches [31, 32], impact the stock market. By extracting information from the stock prices, event window studies represent an intellectual forefather of our proposed technique, which extracts information from insurance prices. However, event window studies are limited to publicly reported events. As a result, they tend to have small sample sizes and questionable relevance beyond listed companies.

Operational risk incidents provide another dataset. Biener et al. [33], Bouveret [34], and Eling et al. [35] analysed different sources of operational losses. Respectively, they found mean losses of \$41M, \$66M,<sup>3</sup> and \$43.49M, as well as median losses of \$1.9M, \$4.7M, and \$1.53M. Bouveret [34] fitted aggregate losses to a spliced distribution in which the body and right tail follow a Lognormal and Generalized Pareto distribution, respectively. Eling et al. [35] fit individual losses to a general linear model with co-variates including firm revenue and industry. These studies [33, 34, 35] are highly dependent on the definition of a cyber incident, which is often applied automatically due to the size of the datasets and are biased toward larger companies who are more likely to report losses.

Romanosky et al. [36] quantify cyber risk using a proprietary dataset with 15,000 incidents. Attack frequencies for each industry are calculated using census data on the number of firms in each industry. Their multivariable regression, based on 265 observations, describes how total cost of incident varies according to an organisation’s revenue, industry, number of records and past incidents. The mean loss figures of References [33–35] are over 10 times larger than a similar figure in Reference [36].

Table 1 summarises a research programme with strong evidence regarding how the number of records in a data breach is distributed and how frequently they occur across the United States. We have estimates [36] of breach frequency for an organisation, although this figure groups all firms in an industry regardless of their size,<sup>4</sup> as

<sup>2</sup><https://www.forbes.com/sites/andygreenberg/2012/08/03/mcafee-explains-the-dubious-math-behind-its-unscientific-1-trillion-data-loss-claim/294dfc4525a3>.

<sup>3</sup>Bouveret restricted his analysis to financial firms and they tend to suffer heavier losses.

<sup>4</sup>Grouping corporations like Target with independent book stores under “retail trade” might explain the particularly low frequency for retail firms.

well as point estimates of financial cost of incidents via survey data [26, 27] and two insightful multivariable regressions based on different datasets [35, 36].

However, quantifying the costs of attacks beyond data breach and regulatory fines is a challenge. In particular, the studies described in Table 1 provide no insights into the potential cost of business email compromise, business interruption incidents, or ransomware attacks. The operational loss studies [33–35] may include such incident types, but these are not differentiated in the analyses.

The next section describes the insurance pricing data we will analyse. It provides insights into policies covering business email compromise, business interruption and ransomware attacks, which have not been empirically studied in the literature so far. Section 4 introduces a method to derive distributions of dollar losses in terms of both frequency and impact, advancing the state-of-the-art in quantifying cyber losses.

### 3 OBSERVED PRICES

This section identifies publicly available cyber insurance data, which will provide context for the framework for inferences introduced in Section 4. Data collection is described in Section 3.1. Section 3.2 provides a quantitative description of cyber insurance prices, building on the overview provided in Reference [39].

#### 3.1 Data Collection

Insurers in the United States operating in admitted markets are required to “file their policies and rate schedules with the state insurance departments” [39]. Rate schedules describe the formulas and tables used to calculate the premium for a given applicant. These documents are made publicly available by the National Association of Insurance Commissioners.

Documents are organised by state and made available in the SERFF Filing Access system. We focused on the state of California, which is the largest in the US, rather than partially search multiple states, to collect the widest range of policies. Even if the set of policies skews toward firms unique to California, the insights from a given policy should have general applicability as there are no differences between states “that would materially bias any results or conclusions,” according to Romanosky et al. [39], who tested this across three states. Downloading filings from others states would lead to an increasing number of repeat filings as many insurers file in multiple states.

Our objective was to quantify how cyber insurance premiums are adjusted according to coverage type, limit, deductible, industry, revenue, and security infrastructure. This will allow us to generate sets of insurance prices for a hypothetical firm with specific characteristics, such as revenue or industry.

We extracted rate schedules related to cyber insurance by searching with keywords “cyber,” “security,” and “privacy,” as in Reference [39]. We downloaded only documents with either a “new program” or “rate” component in the filing type, because these relate to pricing, whereas some filings concern the policy wording and have a “form” or “rule” filing type.

This resulted in 131 unique filings that were narrowed down to 26 rate schedules appropriate for our purposes. Romanosky et al. [39] identified three types of pricing: flat rate pricing, base rate modification, and information security pricing. There were 40 filings related to the first type and they were excluded, because the inflexible pricing structure makes inferences difficult. Further, being sold alongside existing products means these endorsements may not be commercially viable alone.

A further five filings relate to the price of excess layers in which insurers offer additional layers of coverage to supplement an existing policy. These tend to be priced as a percentage of the original policy, which makes inferences difficult. We also excluded filings for policies with pricing structures for specific industries. For example, one policy was priced according to the number of doctors employed by a healthcare provider.

Some filings introduced new coverage areas without updating the original rates. We considered these to be additions to the original filing. A small number of insurers updated prices for the same coverage and these were

Table 2. A Subset of the Baserates and Multiplicative Factors Found in a Rate Schedule, Which Can Be Downloaded from <https://tylermoore.utulsa.edu/cyberschedex.pdf>

Revenue	Base rate	Deductible	Factor	Limit	Factor	Hazard Class	Factor
\$10M or less	\$1,914	\$10K	1	\$500K	0.809	1	.804
\$10M–\$20M	\$2,603	\$25K	.95	\$1M	1	2	1
\$20M–\$50M	\$3,502	\$50K	.89	\$2M	1.132	3	1.497
\$50M–\$100M	\$5,225	\$100K	0.82	\$3M	1.245	4	1.905

Price = (Base rate) × (Deductible Factor) × (Limit Factor) × (Hazard Class Factor).

considered to be new filings. This resulted in 26 unique filings, which correspond to sets of prices, along with metadata including when the pricing scheme was first filed.

A rate schedule<sup>5</sup> does not provide prices directly. We had to read the document explaining how prices are calculated and extract the corresponding tables of multiplicative factors. The prices from the 26 rate schedules are determined by the product of a base rate (in USD) and many multiplicative factors that increase or decrease the price.

After extracting the tables, we can calculate the price-limit-deductible triples for a hypothetical firm. The hypothetical firm would have characteristics, such as revenue or industry, corresponding to each multiplicative factor. Using the factors in Table 2, one of the triples for a retail firm with revenue \$50M would be (4,964, 1,000,000, 25,000), because  $4,964 = 5,225 \times 1 \times 0.95 \times 1$  where 1, 0.95, and 1 are the factors for the limit of 1M, deductible of 25K, and hazard class of 2, respectively.

Generating the dataset consists of taking all combinations of limits and deductibles offered by an insurer then computing the corresponding premium. If a rate schedule provided a choice of 8 deductible amounts and 15 limits, then there would be  $8 \times 15 = 120$  triples. This leads to a total dataset of 6,828 price-limit-deductible triples across all 26 rate schedules, with price varying based on the hypothetical firm’s characteristics. Section 4 introduces a method to infer loss distributions based on how a change in the coverage amount affects the price across this set of triples.

The next subsection illustrates how the multiplicative factors vary with a change in coverage type, limit, deductible, and revenue. It also provides insights into how prices have changed over time.

### 3.2 Quantitative Analysis

We look at how prices are adjusted for coverage type, limit and deductible, and revenue and industry in turn. We then provide a longitudinal perspective on cyber insurance prices.

*Pricing by coverage type.* Table 3 lists the coverage categories identified, along with frequency of occurrence (**n**), along with the mean, standard deviation, minimum, and maximum of the price in dollars. Prices for different coverages are sometimes expressed as a fraction of the cyber liability premium, often in a range (e.g., 0.05–0.15), in which case we used the mid-point.

The prominence of cyber liability coverage is not surprising given most filings fall under insurance lines “related [to] corporate liability policies” [39]. Market entrants began offering coverage including first party, business interruption and ransomware in later years.

Data breach and first party costs vary across insurers in terms of what is covered, often including some combination of the last five entries in Table 3, which explains the high standard deviation. Further, policyholders can combine notification costs, public relations and forensics to build the equivalent of a comprehensive data breach policy. This may explain the seemingly small amount of insurers offering first-party data breach coverage.

<sup>5</sup>An example of which can be downloaded from <https://tylermoore.utulsa.edu/cyberschedex.pdf>.

Table 3. Available Endorsements to Cyber Liability Coverage

Coverage	n	Mean (\$)	Std dev	Min (\$)	Max (\$)
Cyber liability	26	7,211	5,079	1,196	21,084
Data breach/first party costs	15	4,221	4,013	896	16,185
Regulatory proceedings	10	1,201	871	95	3,183
Website multimedia	13	3,188	3,615	190	11,283
Business interruption	14	2,304	2,042	437	7,782
Contingent business interruption	3	438	194	226	606
Ransomware	14	843	535	192	1,862
Wire transfer	10	730	619	157	2,108
Notification Costs	6	2,310	1,182	974	3,656
Crisis management	8	895	727	97	1,763
PCI costs	5	1,392	918	193	2,637
Forensics	2	1,306	604	879	1,733
Data recovery	10	1,236	1,449	97	4,669

Mean, standard deviation, minimum, and max are taken across the  $n$  base premiums for a given coverage type.

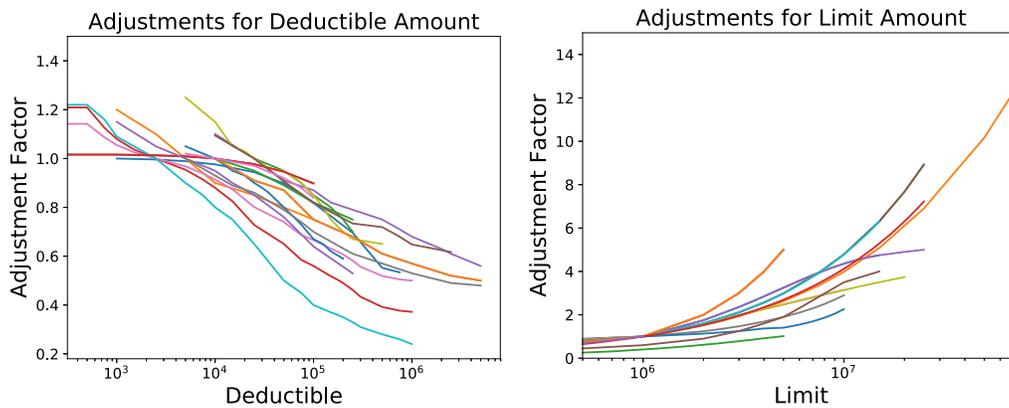


Fig. 1. Insurers apply an adjustment factor depending on the selected deductible (left) and limit (right). Colours are randomly generated and correspond to a different pricing scheme.

Although only three and five insurers offer a forensics and PCI costs endorsement respectively, coverage may still be offered under a general “first-party” costs endorsement.

Wire transfer fraud coverage is consistently priced at around 5–15% of cyber liability. Ransomware coverage tends to be somewhat more expensive. Business interruption coverage is even more expensive still. Variance in the price of multimedia coverage could result from it being a type of third-party cover, which is traditionally difficult to price.

*Pricing by coverage amount.* Figure 1 (left) shows that increasing the deductible leads to a decrease in price as we would expect. The absolute adjustment depends on the specific insurer’s baseline, but we can see exponential increases in the deductible lead to linear decreases in the adjustment factor in general. Insurers use different baseline deductibles (corresponding to an adjustment factor of 1), while most insurers use \$1,000,000 as the baseline limit, hence why most of the lines in Figure 1 (right) pass through (1,000,000, 1.0).

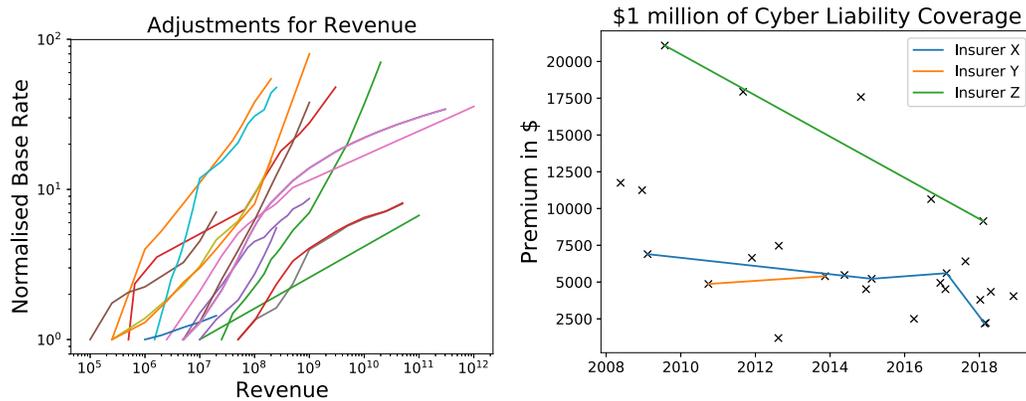


Fig. 2. Insurers apply an adjustment factor depending on the policyholder’s revenue (left); cyber liability insurance premiums over time for selected insurers (right).

The most obvious difference between insurers occurs when they adjust for higher limits. Remarkably, one insurer began including an option for a \$1 billion limit in 2017, which is 56 times the price of a \$1 million limit. Figure 1 (right) shows that some insurers offer sub-linear increases in price for an exponential increase in revenue, which suggests losses exceeding the limit become increasingly less likely.

Some insurers use the applicant’s revenue to set the adjustment for the deductible—when this was the case we used the adjustment for the smallest revenue band available for these figures. Other than these insurers, adjustments for the deductible and limit do not depend on the coverage type or firm characteristics. The importance of this will be established in Section 4, because it suggests the coverage type or industry has little effect on the shape of the distribution of losses.

*Pricing by policyholder characteristics.* We cannot hope to describe in aggregate the range of adjustments for policyholder characteristics. Many of these adjustments are at the underwriter’s discretion, often ranging from 0.75 to 1.25. This is further complicated by insurers describing what they are adjusting for differently, which impedes cross-comparison. We would have to justify why adjustments for “privacy controls,” “encryption,” and “encryption for network data, laptops and mobile devices” refer to the same underlying firm characteristic.

One might expect to be able to compare across adjustments for the policyholder’s industry, especially given there are popular standards for defining industry. As Romanosky et al. [39] observed, “there was no consistency regarding approach [to industry definitions], or any consensus on what the insurance industry would consider the *most* risky.” As a result, we cannot compare across these adjustments, because there is no consistent definition.

Fortunately, we *can* examine how prices are adjusted for revenue. Larger companies are charged higher premiums by assigning tiered base rates according to revenue. Figure 2 (left) displays the adjustments that are made according to revenue. This also shows the maximum revenue that insurers are willing to price without further consultation. It should be noted that most rate schedules offer rates for larger organisations by request.

These adjustments cast further doubt over aggregate statistics regarding expected losses that group together companies regardless of revenues. Some insurers force large companies to pay 100 times the premium that smaller companies do, suggesting the expected loss is up to 100 times as large.

*Pricing over time.* Base rates are typically given for \$1,000,000 of cyber liability coverage, with different rates depending on the organisation’s revenue band. Figure 2 (right) shows the base rate for a retailer with \$50 million of revenue, along with the date the policy was first filed with the regulator.

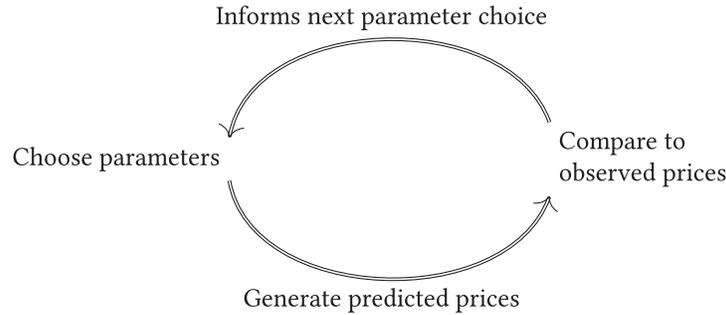


Fig. 3. High level description of our iterative method for inferring loss distributions from insurance prices.

Three insurers updated their rate schedules with new prices, providing the only *apples to apples* comparison, because the prices relate to the same underlying policy. Insurer X updated their prices three times with a notably sharp decrease in 2018. Meanwhile, Insurer Y had a minor increase in price. Insurer Z reduced their price by over 40% after 9 years. It is unclear whether prices for new and existing customers gradually fell during this period.

The trend in prices among market entrants is more difficult to assess due to subtle differences in the products being sold (although these differences are often overstated [39]). The cheapest policy was introduced in 2012 and the maximum limit was \$5M, which suggests it was targeted at smaller companies. We would need to control for changes in coverage to make reliable claims about how prices change over time.<sup>6</sup> Nevertheless, it is fair to say market entrants and price updates have become more frequent.

#### 4 METHOD FOR INFERRING LOSS DISTRIBUTIONS

We now consider how to make inferences from pricing data. Figure 3 describes our iterative method for inferring loss distributions. The method begins by choosing parameters for a given loss distribution. The parameterised loss distribution is used to generate the predicted price for an insurance policy with a given limit and deductible. The set of predicted prices is compared to the observed prices. The result of this comparison is used to improve the next parameter choice. The cycle repeats until some termination condition is met.

We describe how we generate predicted prices from a parameterised loss distribution in Section 4.1. A metric to compare predicted prices with observed prices is defined in Section 4.2. The heuristic governing parameter choices and the termination condition is described in Section 4.3. Finally, Section 4.4 explains how to translate between loss distributions for different coverage types and revenues.

##### 4.1 Generating Price Predictions

This subsection assumes we already have a parameterised distribution for all loss events covered by a given insurance policy. This gives rise to the universe of possible losses  $\Omega = \mathbb{R}_+$ , which is distributed according to a random variable  $X$ . Each possible loss value  $x$  is a non-negative real number occurring with probability determined by the probability density function

$$f(x) : \mathbb{R}_+ \rightarrow \mathbb{R}_+.$$

The probability that the loss amount is greater than  $a$  and less than  $b$  is given by

$$P(a < X < b) = \int_a^b f(x)dx. \tag{1}$$

<sup>6</sup>Anecdotally, industry insiders suggest coverage has become broader and a recent study showed coverage has broadened with reference to war clauses [40]. If this is true, then Figure 2 suggests prices are falling, which would be consistent with greater competition.

The first axiom of probability follows provided  $f$  is non-negative. The second follows by normalising  $f$  so that

$$\begin{aligned} P(\Omega) &= \int_0^{\infty} f(x)dx. \\ &= 1 \end{aligned} \quad (2)$$

The third follows from the definition of an integral.

For our purposes, an insurance contract is a promise that the insured will be indemnified for any losses greater than the deductible  $D$  up to a limit  $L$ . This gives rise to an indemnity function  $I_{D,L}(x) : \mathbb{R}_+ \rightarrow \mathbb{R}_+$  with

$$I_{D,L}(x) = \begin{cases} 0, & \text{for } x < D \\ x - D, & \text{for } D \leq x \leq L + D, \\ L, & \text{for } L + D < x \end{cases} \quad (3)$$

which describes the amount the insurer will pay to the insured for a given loss  $x$ .

As a result, the insurer's expected loss is given by

$$\begin{aligned} E(X)_{D,L} &= \int_{-\infty}^{\infty} I_{D,L}(x)f(x)dx \\ &= \int_D^{L+D} (x - D)f(x)dx + \int_{L+D}^{\infty} Lf(x). \end{aligned} \quad (4)$$

Our method assumes insurance prices are determined by the value of  $E(X)_{D,L}$ . Algebraically we assume that a premium  $p_{D,L}$  with limit  $L$  and deductible  $D$  is given by

$$p_{D,L} = \lambda E(X)_{D,L}, \quad (5)$$

where  $\lambda \geq 1$  is the loading factor. The loading factor determines how much the insurer sets aside for non-claims related expenses. An actuarially fair policy occurs if  $\lambda = 1$ .

## 4.2 Evaluating Loss Distributions via Predicted Prices

This subsection assumes the loss distribution has generated a set of predicted prices. We want to evaluate how the predicted prices compare to observed prices. We define the following cost functions.

*Definition 4.1 (Observed Price Cost Functions).* For a set of premium-limit-deductible triples  $S = \{(p_{D_1,L_1}, L_1, D_1), \dots, (p_{D_n,L_n}, L_n, D_n)\}$ , denote the predicted price and observed price respectively as

$$P_i = E(X)_{D_i,L_i}$$

and

$$O_i = p_{D_i,L_i}.$$

Then define the following cost functions to evaluate a parameterised loss distribution  $X$  with an associated probability density function  $f(x) : \mathbb{R} \rightarrow \mathbb{R}_+$  over the set of prices  $S$ :

$$\begin{aligned} C_T(f|S) &= \frac{1}{n} \sum_{i=1}^n \frac{P_i - O_i}{O_i} \\ C_A(f|S) &= \frac{1}{n} \sum_{i=1}^n \frac{|P_i - O_i|}{O_i}. \end{aligned}$$

The mean of absolute percentage error  $C_A$  sums the absolute differences between predicted and observed prices. A value of 0 occurs if and only if expected prices perfectly predict observed prices. Meanwhile, the mean of percentage error  $C_T$  allows under-predictions to cancel out over-predictions in the aggregate. A score of zero can be achieved by balancing over- and under-predictions.

### 4.3 Choosing Parameters and Termination

Our method can be considered as an iterative optimisation problem. For a distribution with  $n$  parameters, this involves searching through  $\mathbb{R}^n$  for the  $n$ -tuple of parameters minimising the cost function. The problem is non-convex with no way of distinguishing local minima from the globally optimal solution. Further, the objective function  $f$  is relatively costly, because it consists of  $2m$  integrals where  $m$  is the number of price-limit-deductible triples offered by the insurer. These integrals will be computed numerically when an analytic solution is not easily obtained (as will often be the case).

Many optimisation techniques could have been applied and may still prove fruitful in future work. The **particle swarm optimisation (PSO)** heuristic [41] consists of multiple candidate solutions moving around the search space. The direction and speed of movement of each candidate is determined by a weighting of each candidate's best known position, the best known position from the entire swarm, and some randomness. The search stops when the candidates converge or after a certain number of iterations. The process is repeated multiple times to decrease the chance of selecting an unsatisfactory solution.

PSO was chosen because it does not require calculating the gradient of the objective function, unlike gradient descent. It requires few assumptions about the search space, which is ideal for exploratory research. Further, multiple particles in the search space reduce the chance of terminating at a local minima. The Pyswarm [42] package was used for implementation.

### 4.4 Accounting for Multiplicative Pricing

This subsection provides a partial answer as to the effect of changing the coverage type, a characteristic about the firm, or even the profit loading factor  $\lambda$ . It could save re-running the optimisation analysis. We first provide the intuition behind the adjustment.

Suppose that we found that  $f(x)$  best explains a set of prices  $S = \{p_1, \dots, p_n\}$ . Then let  $aS = \{ap_1, \dots, ap_n\}$  be a set of prices for a new coverage type with  $0 < a$ , then intuitively we might say that all losses are  $a$  times as likely. The following reasoning suggests  $af(x)$  explains  $aS$  as well as  $f(x)$  explains  $S$ .

$$C_A(af|aS) = \frac{1}{n} \sum_{i=1}^n \frac{|P_i - ap_i|}{ap_i} = \frac{1}{n} \sum_{i=1}^n \frac{|E(af)_{D_i, L_i} - ap_i|}{ap_i}. \quad (6)$$

Then

$$\begin{aligned} E(af)_{D, L} &= \int_D^{L+D} (x - D)af(x)dx + \int_{L+D}^{\infty} Laf(x) \\ &= aE(f)_{D, L} \end{aligned} \quad (7)$$

substituted into Equation (6) reveals

$$C_A(af|aS) = \frac{1}{n} \sum_{i=1}^n \frac{a|E(f)_{D_i, L_i} - p_i|}{ap_i} = C_A(f|S). \quad (8)$$

For example, if the price of business interruption is 36% of cyber liability across all limits and deductibles, then denoting the cyber liability prices as  $S_{CL}$  we have

$$C_A(0.36f(x)|0.36S_{CL}) = C_A(f(x)|S_{CL}),$$

where  $C_A(f|S_{CL})$  is the cost function value of  $f(x)$ , the cyber liability loss distribution.

Unfortunately,  $af(x)$  is not a probability density function, since

$$\int_0^{\infty} af(x)dx = a \int_0^{\infty} f(x)dx = a \neq 1.$$

Providing  $a < 1$ , we can adjust  $af(x)$  so that a loss of 0 occurs with probability  $1 - a$ . Constructing “mixed discrete/continuous distributions” has precedent in actuarial science [43]. This translates into a business interruption loss of 0 occurring with frequency 0.64. If  $a > 1$ , then re-running the method on the new prices might be a better option. The reader should note that  $af(x)$  provides *as good* a fit for  $\{ap_i\}$  as  $f$  does for  $\{p_i\}$ , but better parameter choices could well be available. Further, this should never be used to adjust for the limit or deductible, because Equation (7) no longer holds.

#### 4.5 Justifying Our Modelling Approach

Our model incorporates a no arbitrage assumption in which insurance prices equal the expected claims pay-out reminiscent of the efficient markets hypothesis. This faces multiple challenges. Insurance markets are not complete and policies are not Arrow-Debreu securities [44], which undermines the theoretical basis of the efficient markets hypothesis. Even complete markets such as stocks and shares have consistently been shown to be inefficient because of market imperfections [45]. The expected insurance payout may further differ from reality due to the lack of available information about cyber losses as demonstrated by Section 2. The resulting uncertainty can be seen in regulatory filings [39] and calls for further research [46]. These disparities between the insurer’s expected distribution of losses and the ground truth distribution can be termed *insurer error*.

*Model error* is introduced by not specifying all relevant factors in translating expectations into premiums. The supply of capital is not a simple function of individual risks. Insurers may strategically under-price cyber insurance relative to expected losses to diversify the portfolio away from natural catastrophe exposure or to build claims data to understand the risk [47, 48]. Accounting for such considerations would require additional model variables, which we have no hope of empirically parameterising.

Assuming insurers set actuarially fair premiums ignores alternative approaches to pricing, such as the variance principle. The observed prices have low limits, which bounds the variance of pay-outs and reduces the difference between the expected value and variance principles. Rather than estimating the loss loading factor or leaving it as another parameter to fit, we set this equal to 1 throughout. This leads to over-estimates as the insurer’s operational costs and profits are treated as an economic loss. This can always be adjusted to the desired loss loading factor using Equation (8) with  $a = 1/\lambda < 1$ , with the resulting parameters having identical model fit.<sup>7</sup>

Admittedly our simplifying assumptions leave inferences vulnerable to insurer, model, and fitting error. Choosing a more complex approach provides no guarantee about eliminating each error source, but adding variables unavoidably reduces parsimony and comprehensibility. Given the reality that *all models are wrong, but some are useful* [49], readers can at least interpret our results as derived from a comprehensible method. More generally, we expect our inferences to complement rather than usurp the results described in Section 2 (of which many have their own validity problems). We recommend for practitioners to assess their risk exposure by triangulating a variety of risk estimates, including those provided by this article.

## 5 EMPIRICAL RESULTS

Our method begins by selecting a loss distribution with unknown parameters to be inferred. The polynomial distribution was chosen for analytic tractability. The Lognormal, Pareto, Burr, and Gamma distributions were selected, because they are commonly used by insurers [50]. Also, they were identified in Section 2. The Weibull distribution was included, because it is mentioned in one of the documents analysed in Section 3.

We generated a set of 6,828 price-limit-deductible triples for cyber liability insurance for a hypothetical retail firm with a revenue of \$50M. We selected a multiplicative factor of 1 for all factors other than limit, deductible, industry, revenue and coverage type. Even if we assumed certain security controls were in place, it would be difficult to assign the corresponding multiplicative factors, because the value depends on the underwriter’s subjective judgement.

<sup>7</sup>Note that if we set  $\lambda > 1$ , the estimates could not be adjusted for a smaller values of  $\lambda$ , since  $a > 1$ .

Table 4. The Distributions and Parameters Values Minimising  $C_A$  across a Set of 2,211 Cyber Liability Prices Offered by One Insurer

Distribution	$f(x)$	Parameter Values	$C_T$	$C_A$
Polynomial	$-\frac{1+a}{c^{a+1}}x^a$	$a = -1.369$ $c = 1.0$	-0.261	0.371
Lognormal	$\frac{1}{\sigma\sqrt{2\pi}}e^{-(\log(x)-\mu)^2/2\sigma^2}$	$\mu = 1e-10, \sigma = 5.418$	-0.312	0.39
Pareto	$\frac{\alpha x_m^\alpha}{x^{\alpha+1}}$	$\alpha = 0.3692, x_m = 0.6712$	-0.266	0.37
Burr	$ck \frac{x^{c-1}}{(1+x^c)^{k+1}}$	$c = 1.219, k = 0.303$	-0.264	0.371
Gamma	$\frac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}$	$\alpha = 0.001579, \beta = 5.02e-08$	-0.058	0.185
Weibull	$\frac{k}{\lambda}(\frac{x}{\lambda})^{k-1}e^{-(\frac{x}{\lambda})^k}$	$k = 0.3039, \lambda = 0.0001574$	-0.263	0.41

Section 5.1 contains analysis of a set of prices from one insurer, enabling a more detailed description of the metrics introduced previously. In Section 5.2, we extend our analysis to consider all of the data collected in Section 3. The County Fair Cyber Loss Distribution is derived in Section 5.3.

### 5.1 Analysis of One Insurer

Before considering the prices from all 26 rate schedules, we focus on the insurer with the most extensive pricing set. This pricing schedule contained 2 211 prices for cyber liability coverage, up to a limit of \$50,000,000.

Table 4 identifies parameter values that minimise  $C_A$  across all of the observed prices. The Gamma distribution does the best job of predicting these prices. The average error ( $C_A$ ) is less than 20% of the observed premium and the net error ( $C_T$ ) is less than 5%. We will achieve as low as 5% absolute error for some insurers in the next subsection.

The relationship between  $C_T$  and  $C_A$  reveals the balance of over- and under-predictions. The Lognormal is the worst offender for consistently under-predicting, as  $C_T$  is close in value to  $C_A$ . This is likely to occur when the distribution is not sufficiently heavy-tailed to predict the prices. Indeed, Burnecki et al. [50] suggest the Pareto is more appropriate than Lognormal “where exceptionally large claims may occur” and the Burr distribution provides a more flexible heavy-tailed distribution.

Both cost functions have an unbounded punishment for over-predictions but a bounded punishment for under-predictions (unless negative prices are predicted). This shifts all of the distributions toward under-prediction. We experimented with a cost function bounding the punishment for over-prediction at +1. For the best performing distributions, there was little difference in the optimal parameter values minimising  $C_A$  and the bounded cost function, and so we omitted this analysis. But it is important to note the parameter choices consistently tilt toward under-prediction.

### 5.2 Market Analysis

We ran the same analysis across all 6,828 of the insurance prices and show the results in Table 5, with each set of prices corresponding to a row. The number of prices ( $n$ ) in a given set is determined by the number of limits and deductibles choices offered by the insurer. The maximum limit ( $L_{max}$ ) ranges from \$1M to 1,000 times that.

No loss distribution consistently outperformed all of the others. This could result from heterogeneity in the sets of observed prices. Factors might include the range of maximum limits and deductibles, or differences in the policy wording. Equally, heterogeneity may result from differing expectations among insurers about losses. In short, the best loss distribution is contingent on the set of prices it aims to predict. So what can we say?

Aggregating the scores suggests the Gamma distribution is the best candidate for predicting cyber liability prices, with the Burr and Weibull distributions performing poorly. However, price set 11 is better explained by

Table 5. The Minimal  $C_A$  Value across for Each Set of Cyber Liability Prices Offered by the Insurers in Our Dataset

Set	Date	n	$L_{max}$	Poly	Lognrm	Pareto	Burr	Gamma	Weibull
1	05/08	30	1,000,000	0.369	0.341	0.235	0.292	<b>0.175</b>	0.229
2	12/08	290	25,000,000	0.358	0.349	0.387	0.393	<b>0.235</b>	0.418
3	02/09	132	1,000,000	0.064	<b>0.055</b>	0.213	0.213	0.125	0.326
4	07/09	168	25,000,000	0.327	0.327	0.389	0.391	<b>0.21</b>	0.384
5	05/10	135	15,000,000	0.178	0.203	0.178	0.178	<b>0.113</b>	0.242
6	09/10	2,211	50,000,000	0.371	0.39	0.37	0.371	<b>0.185</b>	0.41
7	09/11	234	20,000,000	<b>0.162</b>	0.167	0.439	0.446	0.229	0.531
8	11/11	56	5,000,000	0.112	0.102	0.22	0.22	<b>0.082</b>	0.282
9	08/12	74	5,000,000	0.194	0.182	0.171	0.168	<b>0.156</b>	0.182
10	08/12	81	5,000,000	0.35	0.45	<b>0.15</b>	0.339	0.243	0.423
11	11/13	30	1,000,000	0.369	0.362	0.217	0.312	<b>0.128</b>	0.189
12	5/14	458	25,000,000	0.17	<b>0.163</b>	0.333	0.342	0.277	0.516
13	10/14	435	50,000,000	0.306	0.28	0.354	0.358	<b>0.227</b>	0.502
14	12/14	54	2,000,000	0.46	0.45	0.336	0.46	<b>0.172</b>	0.35
15	2/15	55	10,000,000	0.149	<b>0.147</b>	0.203	0.203	0.153	0.314
16	4/16	90	25,000,000	0.19	0.237	0.19	0.19	<b>0.156</b>	0.305
17	9/16	55	5,000,000	<b>0.1</b>	0.101	0.276	0.287	0.183	0.385
18	12/16	396	15,000,000	0.174	<b>0.172</b>	0.313	0.317	0.544	0.465
19	2/17	98	10,000,000	0.213	0.21	0.3	0.314	<b>0.194</b>	0.378
20	2/17	39	2,000,000	0.322	0.31	0.188	0.324	<b>0.174</b>	0.334
21	8/17	168	15,000,000	0.359	0.367	0.341	0.359	<b>0.281</b>	0.375
22	2/18	230	1,000,000,000	<b>0.267</b>	0.326	0.436	0.455	0.45	0.72
23	2/18	325	10,000,000	0.305	0.336	0.26	0.303	<b>0.208</b>	0.487
24	3/18	374	10,000,000	0.286	<b>0.269</b>	0.398	0.417	0.324	0.507
25	4/18	490	25,000,000	0.251	0.25	0.258	0.267	<b>0.213</b>	0.367
26	12/18	120	5,000,000	0.713	0.766	0.293	0.689	0.568	<b>0.179</b>
Mean				0.274	0.279	0.28	0.326	0.222	0.363
Variance				0.138	0.148	0.083	0.111	0.115	0.106

The best score is in bold.

a Weibull parameterisation with a heavy tail. The actuarial model behind the 11th set of prices might expect a relatively more heavy-tailed distribution of losses.

Seeing how these scores correlate with each other sheds some light. The scores in Table 6 tilting toward positive suggests that some sets of prices are easier to predict than others. The Lognormal's performance on a given set of prices is a remarkably good indicator of the polynomial distribution's<sup>8</sup> performance. If the Weibull predicts a set of prices relatively well, then we can expect the Polynomial/Lognormal to do relatively poorly. This provides more evidence of differing expectations regarding how heavy-tailed losses are.

### 5.3 The County Fair Cyber Loss Distribution

The previous subsection provides 26 potential candidates for the parameterised distribution of cyber losses. An argument could be made that the set of prices with the highest maximum limit is most useful, as this leads to

<sup>8</sup>The authors realised the Polynomial distribution is also known as a Power Law distribution during the corrections phase. We could not re-write the analysis in light of this.

Table 6. Describing How the Optimal  $C_A$  Score for One Distribution Correlates with the Optimal  $C_A$  Score of Another Distribution across All of the Rate Schedules ( $n = 26$ )

	Polynomial	Lognormal	Pareto	Burr	Gamma	Weibull
Polynomial	—	0.973	0.28	0.595	0.34	-0.017
Lognormal	0.981	—	0.258	0.607	0.389	0.048
Pareto	0.188	0.146	—	0.817	0.656	0.707
Burr	0.756	0.75	0.64	—	0.736	0.567
Gamma	0.422	0.458	0.449	0.685	—	0.668
Weibull	-0.194	-0.146	0.688	0.283	0.374	—

The top triangle shows the Spearman’s rank correlation coefficient and the bottom triangle shows the Pearson correlation coefficient.

Table 7. An Overview of the Distribution of Losses for the County Fair Cyber Loss Distribution, as Well as the Average Contribution from Each of the Distributions

	County Fair	Poly	Lognorm	Pareto	Gamma	Weibull
$P(\$0 < X < \$10K)$	0.9146	0.7894	0.9021	0.9968	0.9612	0.6374
$P(\$10K < X < \$50K)$	0.0386	0.1422	0.0581	0.0017	0.0124	0.0007
$P(\$50K < X < \$100K)$	0.0094	0.0257	0.0143	0.0004	0.0052	0.0003
$P(\$100K < X < \$250K)$	0.0089	0.0195	0.0121	0.0004	0.0065	0.0004
$P(\$250K < X < \$500K)$	0.0048	0.0084	0.0055	0.0002	0.0043	0.0003
$P(\$500K < X < \$1M)$	0.0035	0.0053	0.0034	0.0001	0.0035	0.0003
$P(\$1M < X < \$2.5M)$	0.0031	0.0041	0.0025	0.0001	0.0034	0.0004
$P(\$2.5M < X < \$5M)$	0.0015	0.0018	0.001	0.0001	0.0017	0.0003
$P(\$5M < X < \$10M)$	0.0009	0.0012	0.0005	0	0.0011	0.0003
$P(\$10M < X < \$50M)$	0.0007	0.0014	0.0005	0.0001	0.0006	0.0007
$P(\$50M < X < \$100M)$	0.0001	0.0003	0.0001	0	0	0.0003
$P(\$100M < X < \$500M)$	0.0001	0.0004	0	0	0	0.0007
$P(\$500M < X < \$1B)$	0	0.0001	0	0	0	0.0003
$P(\$1B < X < \$10B)$	0.0001	0.0001	0	0	0	0.0011
$E(X)_{min}$ (\$)	107 328	242 592	28 360	17 449	23 660	1 3107 18
$E(X)_{mid}$ (\$)	428 261	914 258	70 540	75 655	51 214	6 300 519
$E(X)_{max}$ (\$)	749 194	1 585 924	112 719	133 860	78 768	11 290 318

the least extrapolation. We might instead select the distribution achieving the lowest  $C_A$  score, even though this discards all but 132 of our data points. Alternatively, we could choose the inferred distribution from the most recent filing as this incorporates market experience and recent information. Instead we take our lead from Francis Galton’s [51] method for estimating the size of an ox by aggregating guesses from attendees at a county fair. A similar method was applied to estimate the number of jelly beans in a jar [11].

The **County Fair Cyber Loss Distribution (CFCLD)** is derived by averaging the optimal parameterised loss distribution for each set of prices for a given firm and coverage type. The rest of this section will illustrate the CFCLD by considering cyber liability losses for a retail firm with revenue of \$50M across all 26 pricing schemes. We could easily derive the CFCLD for a different revenue, firm size or coverage type.

Table 7 displays the probability of different loss amounts according to this CFCLD. Treating 0 as part of a continuous distribution means these loss distributions underestimate the proportion of firms facing no losses. Interpreting smaller losses (such as those less than \$50K) as 0 may correct for this. This adjustment suggests an incident rate of 0.0468 per year, since 95% of losses are less than \$50K.

The probability of a loss of between \$100K and \$250K is 0.0089. This falls to 0.0083 for losses between \$250K and \$1M. The probability of a loss of \$1M–\$10M is 0.0045, falling to 0.0009 for \$10–100M.

Inferences about losses beyond the maximum limit are essentially extrapolation. The indemnity payment for a loss exceeding the limit by a dollar is the same as for a loss exceeding the limit by a billion dollars. This leads to a question regarding losses exceeding \$50M as only one firm provides limits beyond this. For the CFCLD, this amounts to 1.4% of the distribution and it is not clear how it should be interpreted.

Table 7 also describes the average contribution to the CFCLD from each distribution. The contributions of each distribution are weighted according to how often each distribution led to the highest  $C_A$  score for a given set of prices. Consequently the Gamma, Lognormal, Polynomial, Pareto, and Weibull distributions contribute 16, 5, 3, 1, and 1, respectively, with no contribution from the Burr distribution.

The implied Pareto distribution is notable for its vanishing tail and it corresponds to the set of prices including the remarkably cheap policy in Figure 2. The Weibull distribution (at least for this parameterisation) is a notable outlier and provides most of the support for the CFCLD's tail. In fact, the singular Weibull parameterisation contributes 99.2% of the losses greater than \$100M despite comprising one 26th of the CFCLD.

Table 7 also includes approximations of the expected loss of each distribution using the buckets in the table. The values of  $E(X)_{min}$ ,  $E(X)_{mid}$ , and  $E(X)_{max}$  are calculated by summing the  $P(a < X < b)$  weighted by the minimum ( $a$ ), midpoint ( $\frac{b-a}{2}$ ), and maximum  $b$  of the range respectively. This presentation allows the reader to understand which sections of each distribution are contributing the most. For example, the over-extrapolated section of the Weibull distribution representing losses of between \$1 billion and 10 billion contributes 54% of  $E(X)_{mid}$ .

Readers might instead calculate expected losses by counting all losses above \$100M as \$100M. They might also decide to omit the contribution of the Weibull distribution. Extracting a concrete expected loss from these distributions is so challenging, because any expected loss relies on some degree of extrapolation, unless the maximum limit exceeds plausible losses.

Thus far we have only considered cyber liability losses, yet we promised estimates for different incident types. Section 4.4 provides a simple way of converting between different coverage types and Section 3 showed that business interruption is priced at 38% of cyber liability. These two results suggest we can multiply all of the probabilities by 0.38 to estimate the distribution of business interruption incidents for the same hypothetical firm. This results in an expected loss of \$154K for business interruption events. The same process for ransomware and wire transfer incidents leads to expected losses of \$64K and \$51K respectively. The estimates could be adjusted for any incident type or hypothetical firm by identifying the corresponding multiplicative factors.

## 6 DISCUSSION

We discuss how the results relates to other attempts to quantify cyber losses in Section 6.1. We critique our method in Section 6.2.

### 6.1 Quantifying Cyber Losses

The results allows us to comment on the shape of the distribution of losses and point-estimates of losses in dollars. Not identifying a single best distribution for cyber losses is in line with data breach studies, which have found no consensus; subsequent studies concluded breaches were best described by power law [17], Lognormal [18], Pareto [19], and log-skew-normal [20] distributions. However, the Gamma distribution being the most likely candidate differs from such studies.

Liability dollar losses could plausibly be less heavy-tailed than distributions of the number of records in a data breach [35]. Liability costs are assigned by courts, which are unlikely to follow the distributions found in data breaches. For example, an equivalent legal ruling to Yahoo!'s<sup>9</sup> breach affecting up to 3 billion accounts is

<sup>9</sup><https://www.reuters.com/article/us-yahoo-cyber/yahoo-says-all-three-billion-accounts-hacked-in-2013-data-theft-idUSKCN1C82O1>.

unlikely as it is an order of magnitude bigger than the next largest. Further evidence can be found in a study [33] of operational losses, which found that “the distribution of the non-cyber risk sample is much heavier tailed than that of the cyber risk” and was corroborated by Reference [35].

Pricing different coverage types by multiplying the price of cyber liability coverage by a constant suggests each type of cyber loss is driven by a similarly shaped distribution. Intuitively this seems wrong, but there is evidence that different incidents are well-modelled by the same distribution. For example, the Lognormal is the best fit for recovery times in IT disruption incidents [22, 23], take-down times for phishing sites [52] and number of records in data breaches [18]. However, Eling et al. [35] show that the Lognormal would significantly over-estimate “actual cyber losses.”<sup>10</sup>

Comparing point-estimates is a simpler task. The lower-bound, mid-point and upper-bound for the expected loss from the CFCLD are \$107K, \$428K, and \$749K respectively. For comparison, one study of 921 event costs has mean of \$7.84M, median of \$250K and maximum of \$750M [36]. All of the operational loss studies [33, 34, 35] found mean losses to be greater than \$40M.

Why are the point estimates from the County Fair Loss Distribution smaller than related studies? The above estimates are average losses given a loss has taken place, whereas we provide the expected loss. Further, our inferences are based on the insurance prices for a retail firm with a revenue of \$50M, whereas losses suffered by larger firms are more likely to find their way into datasets based on publicly reported events. This biases the sample of References [33–36] toward larger losses.<sup>11</sup> The firms likely have revenues exceeding \$50M. This highlights the problem of granularity in cyber security data.

## 6.2 Reflecting on the Method

If our aim is to uncover the true loss distribution, then there are two types of error: insurer error and method error. Insurer error is driven by the insurer’s uncertainty around the loss distribution faced by an insured party. Method error is introduced by making flawed inferences from the observed prices. We can only seek to reduce method error, but it is worth discussing both.

*Insurer Error.* Insurer error emerges from uncertainty in quantifying cyber risk (to be contrasted against certainty in quantifying the risk of an event conditioned on a coin toss). This results in Knightian “unmeasurable” uncertainty [53] as evidenced by discussions in which insurers bemoan the lack of actuarial data in cyber insurance [54, 55]. Even worse than random uncertainty, insurers may exhibit systemic bias resulting from tight professional networks. The resulting group-think could prevent aggregating estimates from mitigating random noise [56]. Finally, the phenomenon of the underwriting cycle, in which prices across all lines of insurance rise and fall cyclically [57], illustrates how actual prices differ from the actuarially fair price across the entire market.

Although the causes of cyber losses are new to insurers, costs are often realised like traditional insurance lines. For example, cyber liability is still assigned by courts and cyber business interruption is calculated via lost revenues. Insurers have amassed much experience quantifying realised losses; the industry dates to at least 14th-century Italy [58] when insurance could be purchased to cover voyages into uncharted territory. Professional qualifications should help with probability estimates given they are improved by even light-weight calibration training interventions [11]. Aggregating the inferred distributions is intended to reduce random noise, much like how different political polls are combined into a *poll of polls* [10].

Prediction markets provide incentives for deviating from group-think. An analogous argument would go: If a line of insurance is systemically over-priced, then individual insurers have an incentive to defect by offering lower prices to increase market share. Figure 2 shows how more market entrants drove some insurers to reduce prices. Although losses should discipline insurers’ generous underwriting, competitive pressures can temporarily

<sup>10</sup>They suggest by a factor of 2 to 12 [35].

<sup>11</sup>Eling et al. [35] use a dataset that excludes losses smaller than \$100K.

lead to systemic under pricing. Cass Sunstein’s aphorism “markets incorporate falsehood as well as truth” [10] summarises this discussion.

*Method Error.* Even if the insurer priced risk perfectly, we might make flawed inferences based on those prices. Results are influenced by the sample of rate schedules and prices. Section 5 shows that the inclusion of rate schedule 11 more than doubled the expected loss from the County Fair Cyber Loss Distribution. For our purposes, choosing the largest dataset was justified, because any anomalies illustrate challenges for future work to overcome. Further, we presented the results so that the reader can extract loss estimates excluding the contribution of anomalous distributions.

Our model assumes risks are considered independently. In reality, insurers must consider how losses correlate with each other and implement costly solvency risk mitigation measures like reinsurance or holding more capital. Not considering insurer risk aversion or wealth endowment could lead to flawed inferences. These factors complicate extracting probabilities from prediction markets [59]. Our method is likely to face even greater challenges given the complexity of insurance markets as compared to the binary options considered by conventional prediction markets.

Given the novelty of the method, it is unclear whether the prediction errors are acceptable. For example, the best performing distribution (the Gamma) across all of the rate schedules has an average error of 22%. These errors do not result from stochastic data generation as the pricing schemes are deterministic. One explanation is that our parsimonious model with two-parameter distributions cannot capture prices in the wild. More complex models or flexible distributions may do so, but they also increase the risk of over-fitting. A separate explanation is that pricing schemes filed with the regulator are strategically “wrong” and that insurers correct this during the underwriter’s subjective adjustment. For example, the default adjustment for each factor may not be 1. Collecting prices quoted to applicants would overcome this issue.

Finally, making the best inference relies on solving an optimisation problem in a non-convex search space. The possibility of terminating at a local minima leads to *fitting error*. Understanding the search space better and selecting the appropriate optimisation heuristic is a must for future work.

## 7 CONCLUSION

We provided empirical observations on how 26 cyber insurance providers in California vary premium by coverage type, amount, policyholder type and over time. The price of business interruption, wire transfer fraud and PCI costs as a percentage of cyber liability coverage are 36%, 12%, and 24%, respectively. Most insurers provide logarithmic increases in premium for a linear increase in the insured’s revenue. The data are inconclusive regarding whether prices are trending downwards, but the rate of market entrance has increased in recent years.

We introduced a method to infer loss distributions from insurance prices. The method uses particle swarm optimisation to iterate through candidate parameter values to identify the parameterised loss distribution that best explains the observed prices. The Gamma, Lognormal, Polynomial, Pareto and Weibull distributions best predicted 16, 5, 3, 1, and 1, respectively, of the 26 sets of premiums.

The County Fair Cyber Loss Distribution aggregates each of the 26 parameterised distributions to provide estimates about cyber losses for a retail firm with a revenue of \$50M. The results suggest the expected loss resulting from cyber liability incidents is \$428K with a 0.006 probability of a loss of between \$1M and \$10M. Expected losses for business interruption, ransomware and wire transfer incidents are \$154K, \$64K, and \$51K, respectively.

These estimates complement existing approaches to quantifying cyber losses by providing distributions of dollar losses, cost estimates for novel incident types, and granular insights for a specific revenue and industry. This first attempt at inferring losses from insurance prices can be improved by speaking to insurance professionals to understand how to construct a better sample of prices, including more flexible distributions to improve predictions, and by analysing performance on prices generated by a known distribution.

A APPENDICES

Table 8. The Parameter Values Corresponding to Table 5

Set	Insurer	Poly	Lognormal	Pareto	Burr	Gamma	Weibull
1	St. Paul Fire and Marine Ins. Co.	-1.44, 124.6	2.485, 5.876	0.1988, 3.06e-05	0.1073, 1.118	0.01903, 9.225e-07	0.08372, 0.004
2	The Travelers Indemnity Co. Of Connecticut	-1.468, 172.3	4.228, 4.426	0.2697, 0.01978	0.6569, 0.4132	0.01359, 2.172e-07	0.0687, 0.001438
3	Hartford Fire Ins. Co.	-1.722, 710.6	4.859, 3.475	0.3338, 2.073	0.8557, 0.3901	0.007281, 4.453e-07	0.3396, 0.0008387
4	Federal Ins. Co.	-1.396, 255.8	4.974, 4.838	0.2031, 0.1322	0.7801, 0.2604	0.01345, 1.018e-07	0.06178, 0.002765
5	Stratford Ins. Co.	-1.367, 1.371	1e-10, 5.452	0.3571, 0.2254	0.5841, 0.6112	0.002467, 1.147e-07	0.491, 0.0004963
6	Travelers Casualty and Surety Co. of America	-1.369, 1.0	1e-10, 5.418	0.3692, 0.6712	1.219, 0.303	0.001579, 5.02e-08	0.3039, 0.0001574
7	Philadelphia Indemnity Ins. Co.	-1.796, 4023.0	7.622, 2.728	0.2625, 3.0	1.036, 0.2534	0.03272, 8.414e-07	0.0433, 0.001744
8	Atlantic Specialty Ins. Co.	-1.66, 548.6	4.612, 3.692	0.3022, 2.511	0.6558, 0.4608	0.008544, 6.02e-07	0.5245, 0.001431
9	Discover Property and Casualty Ins. Co.	-1.286, 2.255	0.3091, 6.4	0.2558, 0.1918	0.1512, 1.494	0.008674, 9.961e-08	0.06174, 0.002714
10	Great American Ins. Co.	-1.469, 1.0	1e-07, 4.339	0.4641, 0.04153	0.1861, 2.393	0.001529, 6.166e-07	0.6957, 0.0002709
11	St. Paul Fire and Marine Ins. Co.	-1.439, 20.87	5.28e-07, 5.914	0.2942, 0.004664	1.043, 0.007721	0.01292, 2.127e-06	0.09196, 0.002301
12	Everest National Ins. Co.	-1.655, 340.9	4.694, 3.515	0.3517, 3.0	0.7278, 0.4834	0.008751, 5.452e-07	0.6507, 0.0005137
13	QBE Ins. Corporation	-1.417, 82.3	3.978, 4.738	0.2743, 2.014	0.699, 0.3914	0.008581, 9.422e-08	0.04431, 0.0007566
14	Zurich American Ins. Co. 18	-1.438, 10.51	0.5467, 5.178	0.325, 0.03928	0.1881, 1.707	0.006347, 9.875e-07	0.08164, 0.001064
15	The Hartford Steam Boiler Inspection and Ins. Co.	-1.706, 262.3	2.881, 3.949	0.3924, 2.878	1.165, 0.3368	0.008694, 1.168e-06	0.0536, 0.0004344
16	'Freedom Specialty Ins. Co.	-1.398, 1.19	1e-10, 5.126	0.3922, 0.3753	0.7442, 0.5269	0.001502, 9.592e-08	0.05443, 0.0002993
17	Arch Ins. Co.	-1.62, 564.9	5.643, 3.364	0.2652, 2.877	1.133, 0.2318	0.02423, 1.238e-06	0.06136, 0.002266
18	AXIS Ins. Co.	-1.655, 308.7	4.411, 3.593	0.3517, 2.501	1.2, 0.2931	0.02773, 3.106e-06	0.06657, 0.0005691
19	The Hartford Steam Boiler Inspection and Ins. Co.	-1.709, 311.7	4.34, 3.406	0.3818, 3.0	0.6891, 0.5541	0.01925, 2.315e-06	0.0584, 0.0005017
20	American Guarantee and Liability Ins. Co.	-1.415, 6.39	0.6926, 5.1	0.329, 0.03881	0.2066, 1.576	0.007277, 1.147e-06	0.08108, 0.001172
21	Starr Indemnity & Liability Co.	-1.307, 1.042	1e-07, 6.028	0.3072, 0.3652	0.6888, 0.446	0.004438, 1.012e-07	0.07249, 0.001049
22	Federal Ins. Co.	-1.478, 104.4	4.655e-07, 6.201	0.3179, 9.962	0.8672, 0.3542	0.008126, 2.027e-07	0.03755, 0.0002793
23	Twin City Fire Ins. Co.	-1.42, 1.0	1e-10, 4.828	0.4142, 0.1765	0.3608, 1.159	0.001997, 2.887e-07	0.8208, 0.0002809
24	North American Specialty Ins. Co.	-1.708, 494.9	4.722, 3.441	0.3449, 3.0	1.032, 0.3336	0.01308, 1.094e-06	0.05877, 0.0006117
25	Key Risk Ins. Co.	-1.477, 20.38	1e-13, 5.427	0.3622, 0.1995	0.6908, 0.5254	0.002696, 1.385e-07	0.7393, 0.0004358
26	Nova Casualty Co.	-1.489, 1.0	1e-10, 4.215	0.3377, 5e-10	0.2231, 2.094	0.003038, 1.892e-07	0.1066, 0.001243

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